Roll No. ....

O. M. R. Serial No. | | | | | | | | |

Question Booklet Number

M. A./M. Sc. (Second Semester) (NEP) EXAMINATION, 2022-23

MATHEMATICS

(Advanced Real Analysis)

Paper Code							
B	0	3	0	8	0	1	T

Questions Booklet Series
A

Time : 1:30 Hours] [*Maximum Marks : 75*

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.
2. The booklet contains 100 questions. Examinee is required to answer 75 questions in the OMR Answer-Sheet provided and not in the question booklet. All questions carry equal marks.
3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to missing or duplicate pages/questions or having any other discrepancy should be got immediately replaced.

(Remaining instructions on the last page)

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।
2. प्रश्न-पुस्तिका में 100 प्रश्न हैं। परीक्षार्थी को 75 प्रश्नों को केवल दी गई OMR आन्सर-शीट पर ही हल करना है, प्रश्न-पुस्तिका पर नहीं। सभी प्रश्नों के अंक समान हैं।
3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR आन्सर-शीट को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका जिसमें कुछ भाग छपने से छूट गए हों या प्रश्न एक से अधिक बार छप गए हों या उसमें किसी अन्य प्रकार की कमी हो, तो उसे तुरन्त बदल लें।

(शेष निर्देश अन्तिम पृष्ठ पर)

(Only for Rough Work)

1. If

$$A = \{\alpha_1, \alpha_2 .... \alpha_n\} \subseteq \mathbf{R},$$

and $m^*(A)$ denotes Lebesgue outer measure of A, then $m^*(A)$ is :

(A) 0

(B) n

(C) less than n

(D) None of the above options

2. If A = $[0,5], B = [3,6]$, then :

(A) $m^*(A) < m^*(B)$

(B) $m^*(A) > m^*(B)$

(C) $m^*(A) = m^*(B)$

(D) $m^*(A) = m^*(B) - 1$

3. If A_1 and A_2 are measurable sets, then $A_1 \cup A_2$ is :

(A) measurable

(B) non-measurable

(C) measurable only if $A_1 \cap A_2 = \phi$

(D) None of the above options

4. If $\Omega = \{a, b, c, d\}, F = \{\phi, \Omega\}$, then :

(A) F is not σ-algebra

(B) F is not algebra

(C) F is σ-algebra

(D) None of the above options

5. If $A \subseteq B$, then which of the following is necessarily true ?

(A) $m^*(A) = m^*(B)$

(B) $m^*(A) > m^*(B)$

(C) $m^*(A) \leq m^*(B)$

(D) None of the above options

6. If I = $[a,b]$ where $a, b \in \mathbf{R}, a < b$, then :

(A) $m^*(I) = b - a$

(B) $m^*(I) = a - b$

(C) $m^*(I) = \infty$

(D) $m^*(I) = -\infty$

7. Let $f : X \to \mathbf{R}$ be a map, X is measurable sets and

$$E = \{x \in X | f(x) > c\},$$

then f is measurable if :

(A) E is non-measurable

(B) E is measurable

(C) E is infinite

(D) None of the above options

8. If A_1 and A_2 are measurable subsets of $[a,b]$, then $A_1 \Delta A_2$ is :

(A) Measurable set

(B) Non-measurable set

(C) Integrable set

(D) None of the above options

9. A subset G = (2, 6] of an interval [1, 6] is .... in [1, 6].

(A) closed

(B) open

(C) Neither open nor closed

(D) Either open or closed

10. If X is a set and if F is a σ-algebra of subsets of X, then which one the following need not be true ?

(A) the empty set $\phi \in F$

(B) $X \in F$

(C) If $A \in F \Rightarrow A^c = X - A \in F$

(D) every singleton set with elements from X is in F.

11. If A_1 and A_2 are measurable subset of $[a,b]$, then :

I – $A_1 \cup A_2$ is measurable

II – $A_1 \cap A_2$ is measurable

(A) Only I is true

(B) Only II is true

(C) Both I and II are true

(D) Both I and II are false

12. A cantor set C ....

(A) is countable

(B) is uncountable and its measure 0

(C) countable and its measure 0

(D) None of the above

13. If A = {1, 2, 3}, F be an algebra on A, $\{1\} \in$ F, then which of the following is necessarily true ?

(A) $\{1,2\} \in F$

(B) $\{1,3\} \in F$

(C) $\{2,3\} \in F$

(D) None of the above

14. If A and B are two sets in F with $A \subseteq B$, then m (A) $\leq$ m (B) (where m is measure). This property is called :

(A) Finite additivity

(B) Countable additivity

(C) Triangle inequality

(D) Monotonicity

15. Let A = $Q^c \cap [0,1]$, then :

(A) $m^*(A) = 1$

(B) $m^*(A) = 0$

(C) $m^*(A) = \frac{1}{2}$

(D) None of the above

16. If A is a set, which of the following is the smallest σ-algebra of subsets of A ?

(A) $\{\phi, A\}$

(B) $\{\phi\}$

(C) $\{A\}$

(D) None of the above

17. If A and B are bounded sets for which $\exists \alpha > 0$ such that $|a-b| \geq \alpha$ for all $a \in A$, and $b \in B$, then :

(A) $m^*(A \cup B) = m^*(A) - m^*(B)$

(B) $m^*(A \cup B) = m^*(A) + m^*(B)$

(C) $m^*(A \cup B) = m^*(A) + m^*(B) - m^*(AB)$

(D) None of the above

18. Consider the following statements :

P : If A is countable set, then $m^*(A) = 0$.

Q : If $m^*(A) = 0$, then A is countable.

Then :

(A) P is true

(B) Q is true

(C) Both P and Q are true

(D) Both P and Q are false

19. If E_1 and E_2 are measurable sets and m is Lebesgue measure, then which of the following is necessarily true ?

(A) $m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1)$

(B) $m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_2)$

(C) $m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$

(D) None of the above

20. Let E be a bounded measurable set of real numbers. Suppose $\exists$ a bounded, countably infinite set of real numbers Ω for which $\{\lambda + E\}_{\lambda \in \Omega}$ is disjoint. Then :

(A) $m(E) \geq 2$

(B) $m(E) \leq -2$

(C) $m(E) = \infty$

(D) $m(E) = 0$

21. Let E be a subset of real numbers. Then :

$$\bigcap_{k=1}^{\infty} \left\{ x \in E \mid f(x) > c - \frac{1}{k} \right\}$$

equals to :

(A) $\{x \in E \mid f(x) < c\}$

(B) $\{x \in E \mid f(x) \geq c\}$

(C) $\{x \in E \mid f(x) < c - 1\}$

(D) None of the above

22. Let $f : [0,1] \to \mathbf{R}$ be a map such that $f(x) = x^2$, then :

(A) f is measurable

(B) f is not measurable

(C) Both (A) and (B) are false

(D) None of the above

23. Which of the following statements is false ?

(A) The outer measure of an interval is its length

(B) Outer measure is translation invariant

(C) Outer measure in finitely additive

(D) Outer measure is not finitely additive

24. Which one of the following is true ?

(A) Outer measure of a singleton set is 1.

(B) Outer measure of a singleton set is 0.

(C) Outer measure of a countable set is ∞.

(D) Outer measure of a finite set is the number of elements in the set.

25. A set E is said to be measurable if :

(A) for each set A,

$$m^*(\mathrm{A}) = m^*(\mathrm{A} \cap \mathrm{E}) + m^*\left(\mathrm{A} \cap \mathrm{E}^c\right)$$

(B) for each set A,

$$m^*(\mathrm{A}) > m^*(\mathrm{A} \cap \mathrm{E}) + m^*\left(\mathrm{A} \cap \mathrm{E}^c\right)$$

(C) for each set A,

$$m^*(\mathrm{A}) < m^*(\mathrm{A} \cap \mathrm{E}) + m^*\left(\mathrm{A} \cap \mathrm{E}^c\right)$$

(D) $m^*(\mathrm{A}) = m^*(\mathrm{A} \cap \mathrm{E}) + m^*\left(\mathrm{A} \cap \mathrm{E}^c\right)$

for some A.

26. Let $\{\mathrm{E}_1, \mathrm{E}_2 ...., \mathrm{E}_n\}$ be a disjoint collection of measurable set. Then :

(A) $m^*\left(\bigcup_{k=1}^{n} \mathrm{E}_k\right) = \sum_{k=1}^{n} m^*(\mathrm{E}_k)$

(B) $m^*\left(\bigcup_{k=1}^{n} \mathrm{E}_k\right) > \sum_{k=1}^{n} m^*(\mathrm{E}_k)$

(C) $m^*\left(\bigcup_{k=1}^{n} \mathrm{E}_k\right) < \sum_{k=1}^{n} m^*(\mathrm{E}_k)$

(D) None of the above

27. Which one of the following is true ?

(A) Lebesgue measure is not countably additive.

(B) Lebesgue measure is countably additive.

(C) Lebesgue measure is not translation invariant.

(D) Labesgue measure assigns the value 0 to the intervals

28. Outer measure is translation invariant, that is, for any set A and number y, then which of the following is true ?

(A) $m^*(\mathrm{A} + y) = m^*(\mathrm{A})$

(B) $m^*(\mathrm{A} + y) > m^*(\mathrm{A})$

(C) $m^*(\mathrm{A} + y) < m^*(\mathrm{A})$

(D) None of the above

29. Consider the following statements :

P : If A and B are disjoint subsets of real numbers then :

$$m^*(A \cup B) = m^*(A) + m^*(B)$$

Q : There are disjoint sets of real numbers A and B such that :

$$m^*(A \cup B) < m^*(A) + m^*(B)$$

Then :

(A) P is true

(B) Q is true

(C) Both P and Q are true

(D) Both P and Q are false

30. Let f be an extended real valued function defined on E and $f^+(x) = \max\{f(x), 0\}$ and $f(x) = \max\{-f(x), 0\}\ \forall x \in E$, then :

(A) f is measurable $\Leftrightarrow$ both f^+ and f^- are measurable

(B) f is measurable $\Leftrightarrow$ f^+ is measurable

(C) f is measurable $\Leftrightarrow$ f^- is measurable

(D) None of the above

31. If for $k = 1, 2, 3,$ $f_k : E \to \mathbf{R}$ are measurable then :

(A) $\max\{f_1, f_2, f_3\}$ is not measurable.

(B) $\max\{f_1, f_2, f_3\}$ is measurable.

(C) $\max\{f_1, f_2\}$ is measurable but $\max\{f_1, f_2, f_3\}$ is not measurable.

(D) $\max\{f_1, f_3\}$ is measurable but $\max\{f_1, f_2, f_3\}$ is not measurable

32. If for K = 1, 2, 3, $f_k : E \to \mathbf{R}$ are measurable, then :

(A) $\min\{f_1, f_2, f_3\}$ is not measurable

(B) $\min\{f_1, f_2\}$ is not measurable

(C) $\min\{f_1, f_2, f_3\}$ is measurable

(D) $\min\{f_1, f_2\}$ is measurable but $\min\{f_1, f_2, f_3\}$ is not measurable.

33. Let f be a continuous map. Then which of the following is necessarily true ?

(A) For any Borel set B, $f^{-1}(B)$ is also a Borel set.

(B) For any Borel set B, $f^{-1}(B)$ is not necessarily a Borel set.

(C) There exists a Borel set B such that $f^{-1}(B)$ is not measurable.

(D) None of the above

34. Let $\mathrm{I} \subseteq \mathbf{R}$ be an interval and $f : \mathrm{I} \to \mathbf{R}$ be a monotonic function. Then :

(A) f is not necessarily measurable

(B) f is measurable

(C) f is measurable only if f is onto

(D) None of the above

35. Which one of the following is false ?

(A) A real valued function that is continuous on its measurable domain is measurable.

(B) A monotonic function that is defined on an interval is measurable.

(C) A monotonic function that is defined on an interval need not be measurable.

(D) Let f be extended measurable real valued function on E and $f = g$, a. e on E, then g is measurable on E.

36. If M is any set, the characteristic function $\mathrm{X_M}$ of the set M is the function given by :

(A) $\mathrm{X_M}(x) = \begin{cases} 1 & \text{if} \quad x \in \mathrm{M} \\ 0 & \text{if} \quad x \notin \mathrm{M} \end{cases}$

(B) $\mathrm{X_M}(x) = \begin{cases} 0 & \text{if} \quad x \in \mathrm{M} \\ 1 & \text{if} \quad x \notin \mathrm{M} \end{cases}$

(C) $\mathrm{X_M}(x) = \begin{cases} 1 & \text{if} \quad x \in \mathrm{M} \\ -1 & \text{if} \quad x \notin \mathrm{M} \end{cases}$

(D) None of the above

37. Let f be a function defined on E and

$$f^{+}(x) = \max\{f(x), 0\},$$

$$f^{-}(x) = \max\{-f(x), 0\},$$

then which one of the following is false ?

(A) If f is measurable on E, then $|f|$ is measurable

(B) If f is measurable on E, then $|f|$ is not measurable

(C) If f^{+} is measurable on E, then for $\mathrm{C} \in \mathbf{R}$ $\mathrm{C}f^{+}$ is measurable

(D) None of the above

38. If $\{f_n\}$ is a sequence of measurable functions on $[a,b]$ such that the sequence $\{fn(x)\}$ is :

(A) Non-measurable function

(B) Measurable function

(C) Not defined

(D) None of the above

39. Every continuous function is :

(A) Non-measurable function

(B) Derivable

(C) Measurable functions

(D) Integrable

40. Let E be a subset of **R** and

$$X_E(x)=\begin{cases}1, & x\in E\\ 0, & x\notin E\end{cases},$$

then for $E_1, E_2 \subseteq \mathbf{R}$ which of the following is true ?

(A) $X_{E_1}(x).X_{E_2}(x)=X_{E_1\cup E_2}(x)$

(B) $X_{E_1}(x).X_{E_2}(x)=X_{E_1}(x)$

(C) $X_{E_1}(x).X_{E_2}(x)=X_{E_1\cap E_2}(x)$

(D) None of the above

41. Let f be a non-negative measurable function on E, then $\int_E f=0$ if and only if :

(A) $f<0$ a. e. (almost everywhere on E)

(B) $f=0$ a. e. on E

(C) $f>0$ a. e. on E

(D) None of the above

42. Let the functions f and g be integrable over E, then for any α and β :

(A) $\int_E(\alpha f+\beta g)=\alpha\int_E f+\beta\int_E g$

(B) $\int_E(\alpha f+\beta g)=\alpha\int_E f-\beta\int_E g$

(C) $\int_E(\alpha f+\beta g)=-\alpha\int_E f+\beta\int_E g$

(D) $\int_E(\alpha f+\beta g)=-\alpha\int_E f-\beta\int_E g$

43. The upper Riemann Integral of f over $[a,b]$ denoted by $(R)\int_a^{-b} f(x)dx$:

(A) $(R)\int_a^{-b} f=\sup.\{L(f,P)|$ P is partition of $[a,b]\}$

(B) $(R)\int_a^{-b} f=\inf.\{U(f,P)|$ p is partition of $[a,b]\}$

(C) $(R)\int_a^{-b} f=\sup.\{U(f,P)|$ p is partition of $[a,b]\}$

(D) $(R)\int_a^{-b} f=\inf.\{L(f,P)|$ p is partition of $[a,b]\}$

44. A bounded real valued function f defined on closed bounded interval $[a,b]$ is Riemann Integrable over $[a,b]$ if :

(A) $(R)\int_{-a}^{b} f<(R)\int_a^{-b} f$

(B) $(R)\int_{-a}^{b} f=(R)\int_a^{-b} f$

(C) $(R)\int_{-a}^{b} f>(R)\int_a^{-b} f$

(D) $(R)\int_{-a}^{b} f\geq(R)\int_a^{-b} f$

45. Let $f : \mathrm{E} \to \mathbf{R}$, $g : \mathrm{E} \to \mathbf{R}$ be non-negative measurable function, then for any $\alpha > 0$ and $\beta > 0$:

(A) $\int_{\mathrm{E}} (\alpha f + \beta g) = \alpha \int_{\mathrm{E}} f$

(B) $\int_{\mathrm{E}} (\alpha f + \beta g) = \beta \int_{\mathrm{E}} g$

(C) $\int_{\mathrm{E}} (\alpha f + \beta g) = \alpha \int_{\mathrm{E}} f + \alpha \int_{\mathrm{E}} g$

(D) $\int_{\mathrm{E}} (\alpha f + \beta g) = \alpha \int_{\mathrm{E}} f + \beta \int_{\mathrm{E}} g$

46. Let $f, g : \mathrm{X} \to \mathbf{R}$ be two non-negative measurable functions with $f \leq g$ on X. Then :

(A) $\int_{\mathrm{X}} f \leq \int_{\mathrm{X}} g$

(B) $\int_{\mathrm{X}} f > \int_{\mathrm{X}} g$

(C) $\int_{\mathrm{X}} f = 0$

(D) $\int_{\mathrm{X}} g = 0$

47. Let :

$$\langle f_1, f_2, f_3, \ldots\ldots f_n, \ldots\ldots\ldots \rangle$$

be a sequence of non-negative measurable functions on E. If $\langle f_n \rangle \to f$ pointwise a. e. (almost everywhere) on E, then :

(A) $\int_{\mathrm{E}} f \leq \liminf \int_{\mathrm{E}} f_n$

(B) $\int_{\mathrm{E}} f > \liminf \int_{\mathrm{E}} f_n$

(C) Both (A) and (B) are true

(D) Neither (a) nor (b) is true

48. Let :

$$f : [a, b] \to \mathbf{R}$$

be a montonic function. Then :

(A) f is continuous on $[a, b]$

(B) f is strictly increasing on $[a, b]$

(C) f is strictly decreasing on $[a, b]$

(D) f is continuous on $[a, b]$ except the set of measure zero.

49. Let $f : \mathrm{E} \to \mathbf{R}$ be an integral function over E. Let X, Y $\subseteq$ E such that X $\cap$Y$=\phi$ and X, Y are measurable. Then :

(A) $\int_{\mathrm{X} \cup \mathrm{Y}} f = \int_{\mathrm{X}} f - \int_{\mathrm{Y}} f$

(B) $\int_{\mathrm{X} \cup \mathrm{Y}} f = \left(\int_{\mathrm{X}} f \right) \left(\int_{\mathrm{Y}} f \right)$

(C) $\int_{\mathrm{X} \cup \mathrm{Y}} f = \int_{\mathrm{Y}} f - \int_{\mathrm{X}} f$

(D) $\int_{\mathrm{X} \cup \mathrm{Y}} f = \int_{\mathrm{X}} f + \int_{\mathrm{Y}} f$

50. Let E be a set of measure zero and define $f(x) = \infty \, \forall x \in \mathrm{E}$ (Assume convention 0. $\infty = 0$) then :

(A) $\int_{\mathrm{E}} f = 0$

(B) $\int_{\mathrm{E}} f = \infty$

(C) $\int_{\mathrm{E}} f > 2$

(D) None of the above

51. Let $f:[0,1]\to\mathbf{R}$ be a map defined as :

$$f(x)=\begin{cases}1, & x\in Q\cap[0,1]\\ -1, & x\in Q^c\cap[0,1]\end{cases}$$

Then :

(A) f is Riemann Integrable

(B) f is not Riemann Integrable

(C) $\cup(f,p)=3\forall$ partition P of $[0,1]$

(D) None of the above

52. Let $f:[a,b]\to\mathbf{R}$ be a montonic function.

Then :

(A) f is Riemann integrable

(B) f is not Riemann Integrable

(C) $\int_a^{-b} f(x)dx \neq \int_{-a}^{b} f(x)dx$

(D) $\int_a^{-b} f(x)dx < \int_{-a}^{b} f(x)dx$

53. Lebesgue integral of the Dirichlet function f defined on $[0,1]$ by :

$$f(x)=\begin{cases}1 & \text{if} \quad x\in[0,1]\cap Q\\ 0 & \text{if} \quad x\in[0,1]\cap Q^c\end{cases}$$

is :

(A) doesn't exist

(B) 0

(C) 1

(D) None of the above

54. Let f be a non-negative measurable function on E, then for any $\lambda>0$:

(A) $m\{x\in E|\, f(x)\leq\lambda\}\geq\frac{1}{\lambda}.\int_E f.$

(B) $m\{x\in E|\, f(x)\geq\lambda\}=\frac{1}{\lambda}.\int_E f$

(C) $m\{x\in E|\, f(x)\geq\lambda\}\geq\frac{1}{\lambda}.\int_E f$

(D) $m\{x\in E|\, f(x)\geq\lambda\}>\frac{1}{\lambda}.\int_E f$

55. Let f be a non-negative measurable function on E, and if E_0 is a subset of E of measure zero, then :

(A) $\int_E f=\int_{E_0} f$

(B) $\int_E f=\int_{E\sim E_0} f-\int_{E_0} f$

(C) $\int_E f=\int_{E\sim E_0} f$

(D) None of the above

56. A non-negative measurable function f on a measurable set E is said to be Integrable over E if :

(A) $\int_E f=0$

(B) $\int_E f<\infty$

(C) $\int_E f=\infty$

(D) None of the above

57. Let the non-negative function f be Integrable over E, then :

(A) f is finite a. e. (almost everywhere) on E.

(B) f is zero a. e. on E

(C) f is constant a. e. on E

(D) None of the above

58. For an extended real valued function f on E, positive part f^+ of f is given by :

(A) $f^+(x) = \max\{-f(x), 0\}$ for all $x \in \text{E}$

(B) $f^+(x) = \max\{f(x), 0\}$ for all $x \in \text{E}$

(C) $f^+(x) = -\max\{f(x), 0\}$ for all $x \in \text{E}$

(D) $f^+(x) = \min\{f(x), 0\}$ for all $x \in \text{E}$

59. For an extended real valued function f on E, negative part f^{-1} of f is given by :

(A) $f^-(x) = \min\{f(x), 0\}$ for all $x \in \text{E}$

(B) $f^-(x) = \max\{f(x), 0\}$ for all $x \in \text{E}$

(C) $f^-(x) = \min\{-f(x), 0\}$ for all $x \in \text{E}$

(D) $f^-(x) = \max\{-f(x), 0\}$ for all $x \in \text{E}$

60. If $|f|$ is Integrable over E, then the Integral of f over E is given by :

(A) $\int_{\text{E}} f = \int_{\text{E}} f^+ - \int_{\text{E}} f^-$

(B) $\int_{\text{E}} f = \int_{\text{E}} f^+ + \int_{\text{E}} f^-$

(C) $\int_{\text{E}} f = \int_{\text{E}} f^- - \int_{\text{E}} f^+$

(D) None of the above

61. Let f be Integrable over E, then :

(A) $\int_{\text{E}} f > \int_{\text{E}\sim\text{E}_0} f$ if $\text{E}_0 \subseteq \text{E}$ and $m(\text{E}_0) = 0$

(B) $\int_{\text{E}} f < \int_{\text{E}\sim\text{E}_0} f$ if $\text{E}_0 \subseteq \text{E}$ and $m(\text{E}_0) = 0$

(C) $\int_{\text{E}} f = \int_{\text{E}\sim\text{E}_0} f$ if $\text{E}_0 \subseteq \text{E}$ and $m(\text{E}_0) = 0$

(D) None of the above

62. Let f be integrable over E and $\{\text{E}_n\}_{n=1}^{\infty}$ a disjoint countable collection of measurable subsets of E whose union is E, then :

(A) $\int_{\text{E}} f < \sum_{n=1}^{\infty} \int_{\text{E}_n} f$

(B) $\int_{\text{E}} f > \sum_{n=1}^{\infty} \int_{\text{E}_n} f$

(C) $\int_{\text{E}} f = \sum_{n=1}^{\infty} \int_{\text{E}_n} f$

(D) None of the above

63. Let f be Integrable over E. If $\{E_n\}_{n=1}^{\infty}$ is an ascending countable collection of measurable subsets of E, then :

(A) $\bigcup_{n=1}^{\infty} \int_{E_n} f = \lim_{n\to\infty} \int_{E_n} f$

(B) $\bigcup_{n=1}^{\infty} \int_{E_n} f < \lim_{n\to\infty} \int_{E_n} f$

(C) $\bigcup_{n=1}^{\infty} \int_{E_n} f > \lim_{n\to\infty} \int_{E_n} f$

(D) None of the above

64. A family F of measurable functions on E is said to be uniformly integrable over E provided :

(A) for each $\in > 0, \exists \delta > 0$ and an $f \in F$, if $A \subseteq E$ is measurable and $m(A) < \delta$, then $\int_A |f| < \in$, $m^*(A) = -2$.

(B) For each $\in > 0, \exists \delta > 0$ such that for each $f \in F$, if $A \subseteq E$ is measurable and $m(A) < \delta$, then $\int_A |f| <' \in$.

(C) There is a $\in > 0$, $\exists \delta > 0$ such that for each $f \in F$, if $A \subseteq E$ is measurable and $m(A) < \delta$, then $\int_A |f| < \in$.

(D) None of the above

65. Let $f : [a,b] \to \mathbf{R}$ be a bounded map such that :

$$\sup_{x\in[a,b]} f(x) = M \quad \inf_{x\in[a,b]} f(x) = m$$

then :

(A) $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$

(B) $\int_a^b f(x)dx > m(b-a)$

(C) $\int_a^{-b} f(x)dx > \int_{-a}^{b} f(x)dx$

(D) None of the above

66. Let $f : [a,b] \to \mathbf{R}$ be a bounded map; P be a partition of $[a,b]$. Let $L(P, f)$ and $U(P, f)$ denote Riemann lower and upper sums respectively, then which of the following is necessarily true ?

(A) $U(P, f) = L(P, f)$

(B) $U(P, f) < L(P, f)$

(C) $U(P, f) \ne L(P, f)$

(D) $U(P, f) \ge L(P, f)$

67. Let :

$$f(x) = \begin{cases} \sin x, & x \in \left[0, \frac{\pi}{2}\right] \\ \cos x, & x \in \left(\frac{\pi}{2}, 4\right] \end{cases}$$

Then which of the following is true ?

(A) f is nowhere continuous

(B) f is Riemann Integrable

(C) f is not Riemann Integrable

(D) None of the above

68. Let :

$$f(x)=\begin{cases} x, & x\in[0,1) \\ x+1, & x\in[1,2) \\ x+3, & x\in[2,3) \end{cases}$$

and D be the set of discontinuity of *f*, then :

(A) $m^*(\mathrm{D})=0$

(B) $m^*(\mathrm{D})<0$

(C) $m^*(\mathrm{D})=3$

(D) $m^*(\mathrm{D})=\infty$

69. Let E be a measurable set and $1\le p\le\infty$. If the functions $f,g\in \mathrm{L}^p(\mathrm{E})$ then :

(A) $f+g\in \mathrm{L}^p(\mathrm{E})$

(B) $f+g\notin \mathrm{L}^p(\mathrm{E})$

(C) Both (A) and (B) are true

(D) None of the above

70. Let E be a measurable set and $1\le p\le\infty$. If the functions $f,g\in \mathrm{L}^p(\mathrm{E})$, then :

(A) $\|f+g\|p\le\|f\|_p-\|g\|_p$

(B) $\|f-g\|p>\|f\|_p+\|g\|_p$

(C) $\|f+g\|p>\|f\|_p+\|g\|_p$

(D) $\|f+g\|p\le\|f\|_p+\|g\|_p$

71. For $1<p<\infty, q$ is the conjugate of p $\left(i.e. \frac{1}{p}+\frac{1}{q}=1\right)$ and any *two* positive numbers *a* and *b* :

(A) $ab>\frac{a^p}{p}+\frac{b^q}{q}$

(B) $ab\ge\frac{a^p}{p}+\frac{b^q}{q}$

(C) $ab\le\frac{a^p}{p}+\frac{b^q}{q}$

(D) None of the above

72. If $f,g\in \mathrm{L}^2(\mathrm{E})$, then which of the following is necessarily true ?

(A) $\int_{\mathrm{E}}(\lambda f+g)^2=\lambda^2\int_{\mathrm{E}} f^2+ 2\lambda\int_{\mathrm{E}} f.g+\int_{\mathrm{E}} g^2 \forall\lambda\in\mathbf{R}$

(B) $\int_{\mathrm{E}}(\lambda f+g)^2<0\ \forall\lambda\in\mathbf{R}$

(C) $\int_{\mathrm{E}}(\lambda f+g)^2=\lambda\int_{\mathrm{E}} f^2+ \int_{\mathrm{E}} g\ \forall\lambda\in\mathbf{R}$

(D) None of the above

73. If f is bounded functions on E and $f\in \mathrm{L}^p(\mathrm{E})$, then :

(A) If $p_2>p_1$, then $f\notin \mathrm{L}^{p_2}(\mathrm{E})$

(B) $\exists\, p_2>p_1$ such that $f\notin \mathrm{L}^{p_2}$ (E)

(C) $\forall\, p_2>p_1, f\in \mathrm{L}^{p_2}(\mathrm{E})$

(D) None of the above

74. Let :

$$f(x) = \log e\left(\frac{1}{x}\right), \forall x \in (0,1]$$

then :

(A) $f \in L^p(0,1] \forall 1 \le p < \infty$

(B) $f \in L^\infty(0,1]$

(C) $\exists p$ such that $1 \le p < \infty, f \notin L^p(0,1]$

(D) None of the above

75. Let ϕ be a convex function on $(-\infty, \infty)$, f an integrable function over $[0,1]$ such that ϕ of is also integrable over $[0,1]$, then which of the following is necessarily true ?

(A) $\phi\left(\int_0^1 f(x)dx\right) > \int_0^1 (\phi o f)(x)dx$

(B) $\phi\left(\int_0^1 f(x)dx\right) = \int_0^1 (\phi o f)(x)dx$

(C) $\phi\left(\int_0^1 f(x)dx\right) \le \int_0^1 (\phi o f)(x)dx$

(D) None of the above

76. Let v be a signed measure on measurable space (X, M). Then :

(A) Every measurable subset of positive set is positive

(B) There exists a measurable subset of positive set which is not positive.

(C) Both (A) and (B) are true

(D) Both (A) and (B) are false

77. Let U be a signed measure on measurable space (X, M). Then there is a positive set A for v and a negative set B for v for which X = $A \cup B$ and $A \cap B = \phi$. This is the statement of :

(A) Jensen's Inequality

(B) Holder's Inequality

(C) The Hahn Decomposition theorem

(D) None of the above

78. Let ν be a signed measure, A is measurable set (w. r. to ν) and $\forall$ measurable set E $\subseteq$ A , ν (E) ≥ 0 then :

(A) A is said to be positive (w. r. to ν)

(B) A is said to be negative (w. r. to ν)

(C) A can never be positive

(D) Both (B) and (C) are true

79. A continuous function f on (a,b) is convex if and only if :

(A) $f\left(\frac{x_1+x_2}{2}\right) > \frac{f(x_1)+f(x_2)}{2}$, $\forall x_1, x_2 \in (a,b)$

(B) $f\left(\frac{x_1+x_2}{2}\right) \le \frac{f(x_1)+f(x_2)}{2}$, $\forall x_1, x_2 \in (a,b)$

(C) $f\left(\frac{x_1+x_2}{2}\right) > \frac{f(x_1)+f(x_2)}{2}$ for some $x_1, x_2 \in (a,b)$

(D) None of the above

80. Let ϕ be twice differentiable map on (a,b), then :

(A) ϕ is convex $\Leftrightarrow$ ϕ'' is non-negative

(B) ϕ is convex $\Leftrightarrow$ ϕ'' is negative

(C) Both (A) and (B) are true

(D) None of the above

81. Let G_δ be countable intersection of open sets. Then which of the following is true ?

(A) G_δ is not measurable.

(B) G_δ is not measurable because open sets are Borel sets.

(C) Each G_δ is measurable.

(D) None of the above

82. Let F_σ be the countable union of closed sets. Then which of the following is necessarily true ?

(A) F_σ is measurable

(B) F_σ is not measurable

(C) F_σ is not measurable because closed sets are not Borel sets

(D) None of the above

83. Consider the following statements :

P : If a set is measurable then it is also a Borel set.

Q : If a set has measure zero then it is also a countable set.

Then :

(A) P is true.

(B) Q is true.

(C) Both P and Q are true.

(D) Both P and Q are false.

84. Let $f : [a,b] \to \mathbf{R}$ be a of bounded map and $\langle P_1, P_2 .... P_n .... \rangle$ be a sequence of partitions of [a, b] such that :

$$\lim_{n\to\infty}\left[U(f, P_n) - L(f, P_n)\right] = 0$$

then :

(A) f is Riemann Integrable over $[a,b]$

(B) f can't be Riemann Integrable over $[a,b]$

(C) $f(x) = 0 \, \forall x \in [a,b]$

(D) $f(x) = f(x+1) \, \forall x \in [a,b]$

85. Let f be Integrable over E and C be a measurable subset of E, χ_c denotes characteristic function.

Then :

(A) $\int_c f = \int_E f \cdot \chi_c$

(B) $\int_c f \neq \int_E f \cdot \chi_c$

(C) $\int_c f > \int_E f \cdot \chi_c$

(D) None of the above

86. Let f be Integrable over **R** and $f = 0$, almost everywhere (a.e) on **R**. Then :

(A) $\int_A f \neq 0$ for some measurable set A

(B) $\int_A f < 0$ for some measurable set A

(C) $\int_A f = 0$ for some measurable set A

(D) None of the above

87. Let f be Integrable over **R** with $\int_A f = 0$ for all measurable sets A. Then :

(A) $f = 0$ almost everywhere on **R**

(B) $f(x) \neq 0 \forall x \in \mathbf{R}$

(C) f can never assume the value '0'

(D) None of the above

88. Let $f = 0$ almost everywhere on **R**. Then :

(A) $\int_0 f = 0$ for every open set 0

(B) $\int_0 f \neq 0$ for every open set 0

(C) $\int_0 f \neq 0$ for any open set 0

(D) None of the above

89. Let f and g be a measurable function on E and

$$h = \frac{1}{2}\left[(f+g)^2 - f^2 - g^2\right]$$

Then :

(A) h is measurable

(B) h can't be measurable

(C) h is measurable only if $m^*(\mathrm{E}) = 0$

(D) None of the above

90. Let I = $[a,b]$ be a closed interval. Then which of the following is true ?

(A) $m^*(\mathrm{I}) = 5$

(B) I is measurable.

(C) I is not measurable.

(D) None of the above

91. Let $f:(1,\infty) \to \mathbf{R}$ be a map such that :

$$f(x) = \frac{\sqrt{x}}{1+\log_e x}, x > 1$$

Then :

(A) $f \in \mathrm{L}^p(\mathrm{E}) \forall p$

(B) $f \in \mathrm{L}^p(\mathrm{E})$ if $p \neq 2$

(C) $f \in \mathrm{L}^p(\mathrm{E}) \Leftrightarrow p = 2$

(D) None of the above

92. If A is measurable set of finite outer measure that is contained in B, then :

(A) $m^*(B \sim A) > m^*(B) - m^*(A)$

(B) $m^*(B \sim A) < m^*(B) - m^*(A)$

(C) $m^*(B \sim A) = m^*(B) - m^*(A)$

(D) $m^*(B \sim A) = m^*(A) + m^*(B)$

93. Which of the following is not true ?

(A) The translate of a measurable is measurable

(B) The translate of a measurable set is need not be measurable

(C) The Borel σ algebra is contained in every σ algebra that contains all open sets.

(D) The Borel σ algebra is the intersection of all the σ algebras of subsets of $\mathbf{R}$ that contains the open sets.

94. If $\{A_k\}_{k=1}$ is asceding collection of measurable sets, then :

(A) $m\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{k \to \infty} m(Ak)$

(B) $m\left(\bigcup_{k=1}^{\infty} A_k\right) = 1 - \lim_{k \to \infty} m(A_k)$

(C) $m\left(\bigcup_{k=1}^{\infty} A_k\right) = 1 + \lim_{k \to \infty} m(A_k)$

(D) None of the above

95. Let $\{E_k\}_{k=1}^{\infty}$ be a countable collection of measurable sets for which $\sum_{k=1}^{\infty} m(E_k) < \infty,$ then :

(A) almost all $x \in \mathbf{R}$ belongs to all $E_{k's}$.

(B) all $x \in \mathbf{R}$ belongs to at most finitely many of the $E_k / E_{k's}$

(C) almost all $x \in \mathbf{R}$ belongs to at most finitely many of the $E_k{}'s$

(D) None of the above

96. Which of the following is not true for the set function of Lebesgue measure ?

(A) For any finite disjoint collection $\{E_k\}_{k=1}^{n}$ of measure set :

$$m\left(\bigcup_{k=1}^{n} E_k\right)=\sum_{k=1}^{n} m(E_k)$$

(B) For any countable collection $\{E_k\}_{k=1}^{\infty}$ of measurable sets that covers a measurable set E :

$$m(E)\geq\sum_{k=1}^{\infty} m(E_k)$$

(C) Ff A and B are measurable sets and $A \subseteq E$ then $m(A)\leq m(B)$.

(D) None of the above

97. Which of the following is false ?

(A) f be an extended measurable real valued function on E and $f = g$ a. e. on E, then g is measurable on E.

(B) Let f be an extended measurable real-valued function on E and $f = g$ a. e. on E, then g need not be measurable on E.

(C) A monotone function that is defined on an interval is measurable.

(D) None of the above

98. Which of the following is false ?

(A) Let f and g be measurable function on E that are finite a. e. on E the fg is measurable on E.

(B) Let f and g be measurable function on E that are finite a. e. on E then fg need not be measurable on E.

(C) Let f and g be measurable function on E that are finite a. e. on E, then $f+g$ is measurable on E.

(D) None of the above

99. Let $f(x)=e^{4x}$. Then :

(A) f is not convex on $[-1,1]$

(B) f is not convex on $[-1, \infty)$

(C) f is convex on (–1, 1)

(D) None of the above

100. Let $f:[0,1]\to\mathbf{R}$ be a map such that $f(x)=x^2$, then :

(A) $\phi\left[\lambda x_1+(1-\lambda)x_2\right]\leq\lambda\phi(x_1)+(1-\lambda)\phi(x_2)\,\forall\lambda\in[0,1]$

(B) $\phi\left[\lambda x_1+(1-\lambda)x_2\right]>\lambda\phi(x_1)+(1-\lambda)\phi(x_2)\,\forall\lambda\in[0,1]$

(C) $\phi\left[\lambda x_1+(1-\lambda)x_2\right]>\lambda\phi(x_1)+(1-\lambda)\phi(x_2)$ for some $\lambda\in[0,1]$

(D) None of the above

4. Four alternative answers are mentioned for each question as—A, B, C & D in the booklet. The candidate has to choose the correct answer and mark the same in the OMR Answer-Sheet as per the direction :

Example :

Question :

Q. 1 Ⓐ ● Ⓒ Ⓓ

Q. 2 Ⓐ Ⓑ ● Ⓓ

Q. 3 Ⓐ ● Ⓒ Ⓓ

Illegible answers with cutting and over-writing or half filled circle will be cancelled.

5. Each question carries equal marks. Marks will be awarded according to the number of correct answers you have.

6. All answers are to be given on OMR Answer sheet only. Answers given anywhere other than the place specified in the answer sheet will not be considered valid.

7. Before writing anything on the OMR Answer Sheet, all the instructions given in it should be read carefully.

8. After the completion of the examination candidates should leave the examination hall only after providing their OMR Answer Sheet to the invigilator. Candidate can carry their Question Booklet.

9. There will be no negative marking.

10. Rough work, if any, should be done on the blank pages provided for the purpose in the booklet.

11. To bring and use of log-book, calculator, pager and cellular phone in examination hall is prohibited.

12. In case of any difference found in English and Hindi version of the question, the English version of the question will be held authentic.

Impt. : On opening the question booklet, first check that all the pages of the question booklet are printed properly. If there is ny discrepancy in the question Booklet, then after showing it to the invigilator, get another question Booklet of the same series.

4. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार सम्भावित उत्तर—A, B, C एवं D हैं। परीक्षार्थी को उन चारों विकल्पों में से सही उत्तर छाँटना है। उत्तर को OMR आन्सर-शीट में सम्बन्धित प्रश्न संख्या में निम्न प्रकार भरना है :

उदाहरण :

प्रश्न :

प्रश्न 1 Ⓐ ● Ⓒ Ⓓ

प्रश्न 2 Ⓐ Ⓑ ● Ⓓ

प्रश्न 3 Ⓐ ● Ⓒ Ⓓ

अपठनीय उत्तर या ऐसे उत्तर जिन्हें काटा या बदला गया है, या गोले में आधा भरकर दिया गया, उन्हें निरस्त कर दिया जाएगा।

5. प्रत्येक प्रश्न के अंक समान हैं। आपके जितने उत्तर सही होंगे, उन्हीं के अनुसार अंक प्रदान किये जायेंगे।

6. सभी उत्तर केवल ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर ही दिये जाने हैं। उत्तर-पत्रक में निर्धारित स्थान के अलावा अन्यत्र कहीं पर दिया गया उत्तर मान्य नहीं होगा।

7. ओ. एम. आर. उत्तर-पत्रक (OMR Answer Sheet) पर कुछ भी लिखने से पूर्व उसमें दिये गये सभी अनुदेशों को सावधानीपूर्वक पढ़ लिया जाये।

8. परीक्षा समाप्ति के उपरान्त परीक्षार्थी कक्ष निरीक्षक को अपनी OMR Answer Sheet उपलब्ध कराने के बाद ही परीक्षा कक्ष से प्रस्थान करें। परीक्षार्थी अपने साथ प्रश्न-पुस्तिका ले जा सकते हैं।

9. निगेटिव मार्किंग नहीं है।

10. कोई भी रफ कार्य, प्रश्न-पुस्तिका के अन्त में, रफ-कार्य के लिए दिए खाली पेज पर ही किया जाना चाहिए।

11. परीक्षा-कक्ष में लॉग-बुक, कैलकुलेटर, पेजर तथा सेल्युलर फोन ले जाना तथा उसका उपयोग करना वर्जित है।

12. प्रश्न के हिन्दी एवं अंग्रेजी रूपान्तरण में भिन्नता होने की दशा में प्रश्न का अंग्रेजी रूपान्तरण ही मान्य होगा।

महत्वपूर्ण : प्रश्नपुस्तिका खोलने पर प्रथमतः जाँच कर देख लें कि प्रश्न-पुस्तिका के सभी पृष्ठ भलीभाँति छपे हुए हैं। यदि प्रश्नपुस्तिका में कोई कमी हो, तो कक्षनिरीक्षक को दिखाकर उसी सिरीज की दूसरी प्रश्न-पुस्तिका प्राप्त कर लें।